Module
7
Design of Springs

Lesson
2

Design of Helical Springs
for Variable Load

At the end of this lesson, the students should be able to understand:

- Nature of varying load on springs
- Modification of Soderberg diagram
- Estimation of material properties for helical spring
- Types of helical springs
- Design considerations for buckling and surge.

7.2.1 Design of helical spring for variable load

In the earlier lecture, we have learned about design of helical springs for static loads. In many applications, as for example in railway carriages or in automobile suspension systems the helical springs used are constantly under variable load. Hence, it is understood that whenever there is a variable load on a spring the design procedure should include the effect of stress variation in the spring wire. The methodology used is the modified Soderberg method. we have learnt about Soderberg method in earlier chapter, here, the necessary modifications applicable to helical spring design will be discussed.

In the case of a spring, whether it is a compression spring or an extension spring, reverse loading is not possible. For example, let us consider a compression spring placed between two plates. The spring under varying load can be compressed to some maximum value and at the most can return to zero compression state (in practice, some amount of initial compression is always present), otherwise, spring will loose contact with the plates and will get displace from its seat. Similar reason holds good for an extension spring, it will experience certain amount of extension and again return to at the most to zero extension state, but it will never go to compression zone. Due to varying load, the stress pattern which occurs in a spring with respect to time is shown in Fig.7.2.1. The load which causes such stress pattern is called repeated load. The spring materials, instead of testing under reversed bending, are tested under repeated torsion.
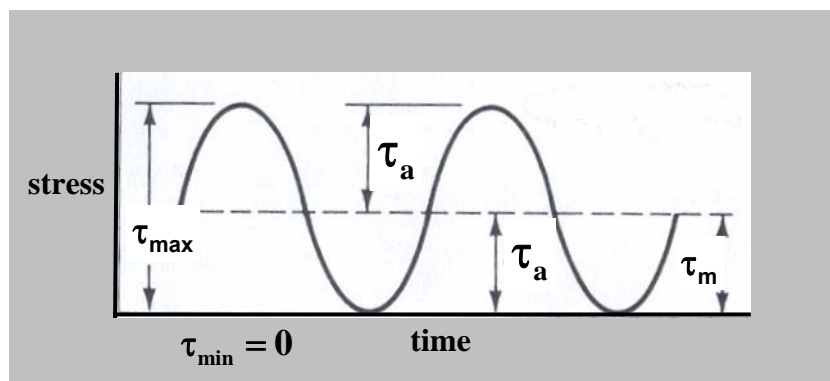

Fig 7.2.1

From Fig.7.2.1 we see that ,

$$\tau_m = \tau_a = \frac{\tau_{max}}{2}$$ (7.2.1)

Where, τ_a is known as the stress amplitude and τ_m is known as the mean stress or the average stress. We know that for varying stress, the material can withstand stress not exceeding endurance limit value. Hence, for repeated torsion experiment, the mean stress and the stress amplitude become,

$$\tau_m = \tau_a = \frac{\tau_{max}}{2} = \frac{\tau_e}{2}$$ (7.2.2)

7.2.1.1 Soderberg failure criterion

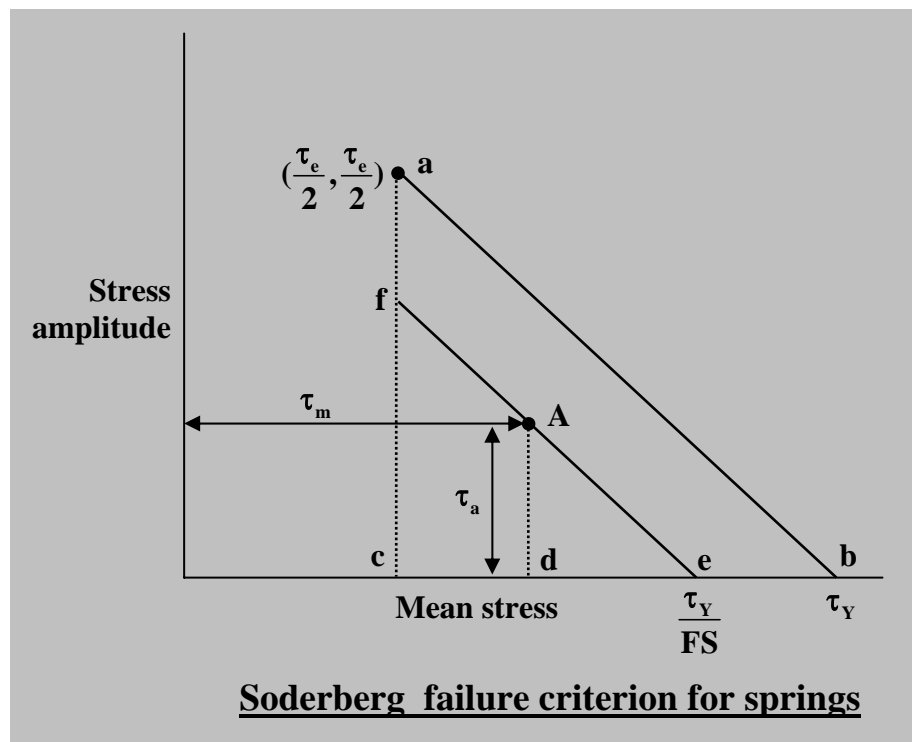The modified Soderberg diagram for repeated stress is shown in the Fig 7.2.2.

Fig 7.2.2

The stress being repeated in nature, the co-ordinate of the point **a** is $\frac{\tau_e}{2}, \frac{\tau_e}{2}$. For safe design, the design data for the mean and average stresses, τ_a and τ_m respectively, should be below the line a-b. If we choose a value of factor of safety (FS), the line a-b shifts to a newer position as shown in the figure. This line e-f in the figure is called a safe stress line and the point A (τ_m, τ_a) is a typical safe design point.

Considering two similar triangles, **abc** and **Aed** respectively, a relationship between the stresses may be developed and is given as,

$$\frac{\tau_a}{\frac{\tau_Y}{FS} - \tau_m} = \frac{\frac{\tau_e}{2}}{\tau_Y - \frac{\tau_e}{2}}$$

(7.2.3)

where τ_Y is the shear yield point of the spring material.

In simplified form, the equation for Soderberg failure criterion for springs is

$$\frac{1}{FS} = \frac{\tau_m}{\tau_Y} + \frac{\tau_a}{\tau_Y}(\frac{2\tau_Y}{\tau_e} - 1)$$

(7.2.4)

The above equation is further modified by considering the shear correction factor, K_s and Wahl correction factor, K_w. It is a normal practice to multiply τ_m by K_s and to multiply τ_a by K_w.

$$\frac{1}{FS} = \frac{K_s\tau_m}{\tau_Y} + \frac{K_w\tau_a}{\tau_Y}(\frac{2\tau_Y}{\tau_e} - 1)$$

(7.2.5)

The above equation for Soderberg failure criterion for will be utilized for the designing of springs subjected to variable load.

7.2.1.2 Estimation of material strength

It is a very important aspect in any design to obtain correct material property. The best way is to perform an experiment with the specimen of desired material. Tensile test experiments as we know is relatively simple and less time consuming. This experiment is used to obtain yield strength and ultimate strength of any given material. However, tests to determine endurance limit is extremely time consuming. Hence, the ways to obtain material properties is to consult design data book or to use available relationships, developed through experiments, between various material properties. For the design of springs, we will discuss briefly, the steps normally used to obtain the material properties.
One of the relationships to find out ultimate strength of a spring wire of diameter **d** is,

$$\sigma_{ut} = \frac{A_s}{d^{m_s}}$$

(7.2.6)

For some selected materials, which are commonly used in spring design, the values of A_s and m_s are given in the table below.

$$A_s \qquad m_s$$

Hard-drawn wire	1510	0.201
Oil-tempered wire	1610	0.193
Chrome-vanadium wire	1790	0.155
Chrome-silicon wire	1960	0.091
Music wire	2060	0.163

The above formula gives the value of ultimate stress in MPa for wire diameter in mm. Once the value of ultimate strength is estimated, the shear yield strength and shear endurance limit can be obtained from the following table developed through experiments for repeated load.

Wire Type	$\dfrac{\tau_e}{\sigma_{ult}}$	$\dfrac{\tau_y}{\sigma_{ult}}$
Hard-drawn wire	0.21	0.42
Oil-tempered wire	0.22	0.45
Chrome-vanadium wire	0.20	0.51
Chrome-silicon wire	0.20	0.51
Music wire	0.23	0.40
302 SS wire	0.20	0.46

Hence, as a rough guideline and on a conservative side, values for shear yield point and shear endurance limit for major types of spring wires can be obtained from ultimate strength as,

$$\frac{\tau_y}{\sigma_{ult}} = 0.40 \qquad \text{and} \qquad \frac{\tau_e}{\sigma_{ult}} = 0.20 \tag{7.2.7}$$

With the knowledge of material properties and load requirements, one can easily utilize Soderberg equation to obtain spring design parameters.

7.2.2 Types of springs

There are mainly two types of helical springs, compression springs and extension springs. Here we will have a brief look at the types of springs and their nomenclature.

7.2.2.1 Compression springs

Following are the types of compression springs used in the design.

(a) Plain ends

        Total coils, N_T   : N

        Solid length, L_S   : $d\,(\,N_T + 1\,)$

        Free length, L   : $L_S + \delta_{max} + \delta_{allowance}$

        Pitch, p   : $(\,L - d\,)\,/\,N$

Plain end spring
Fig 7.2.3

In the above nomenclature for the spring, **N** is the number of active coils, i.e., only these coils take part in the spring action. However, few other coils may be present due to manufacturing consideration, thus total number of coils, **N_T** may vary from total number of active coils.

Solid length, **L_S** is that length of the spring, when pressed, all the spring coils will clash with each other and will appear as a solid cylindrical body.
The spring length under no load condition is the free length of a spring. Naturally, the length that we visualise in the above diagram is the free length.

Maximum amount of compression the spring can have is denoted as δ_{max}, which is calculated from the design requirement. The addition of solid length and the δ_{max} should be sufficient to get the free length of a spring. However, designers consider an additional length given as $\delta_{allowance}$. This allowance is provided to avoid clash between to consecutive spring coils. As a guideline, the value of $\delta_{allowance}$ is generally 15% of δ_{max}.

The concept of pitch in a spring is the same as that in a screw.

(b) Plain and Ground ends

        Total coils, N_T   : N + 1

        Solid length, L_S   : $d\,(\,N_T\,)$

        Free length, L $L_S + \delta_{max} + \delta_{allowance}$

        Pitch, p   : L / (N + 1)

Plain and Ground end spring
Fig 7.2.4

The top and bottom of the spring is grounded as seen in the figure. Here, due to grounding, one total coil is inactive.

(c) Squared or closed ends

Total coils, N_T : N + 2

Solid length, L_S : $d (N_T + 1)$

Free length, L $L_S + \delta_{max} + \delta_{allowance}$

Pitch, p   : (L - 3d) / N

Squared or closed end spring

Fig 7.2.5

In the Fig 7.2.5 it is observed that both the top as well as the bottom spring is being pressed to make it parallel to the ground instead of having a helix angle. Here, it is seen that two full coils are inactive.

(d) Squared and ground ends

Total coils, N_T : N + 2

Solid length, L_S : $d (N_T)$

Free length, L $L_S + \delta_{max} + \delta_{allowance}$

Pitch, p   : (L - 2d) / N

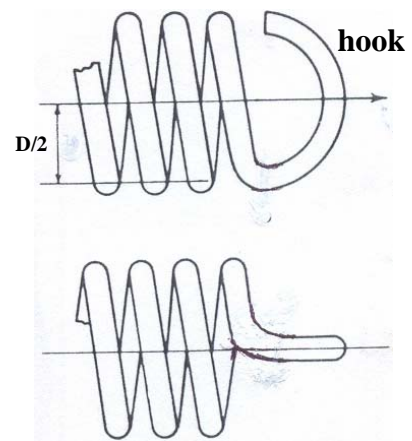**Squared and ground end spring**

Fig 7.2.6

It is observed that both the top as well as the bottom spring, as earlier one, is being pressed to make it parallel to the ground, further the faces are grounded to allow for proper seat. Here also two full coils are inactive.

7.2.2.2 Extension springs

Part of an extension spring with a hook is shown in
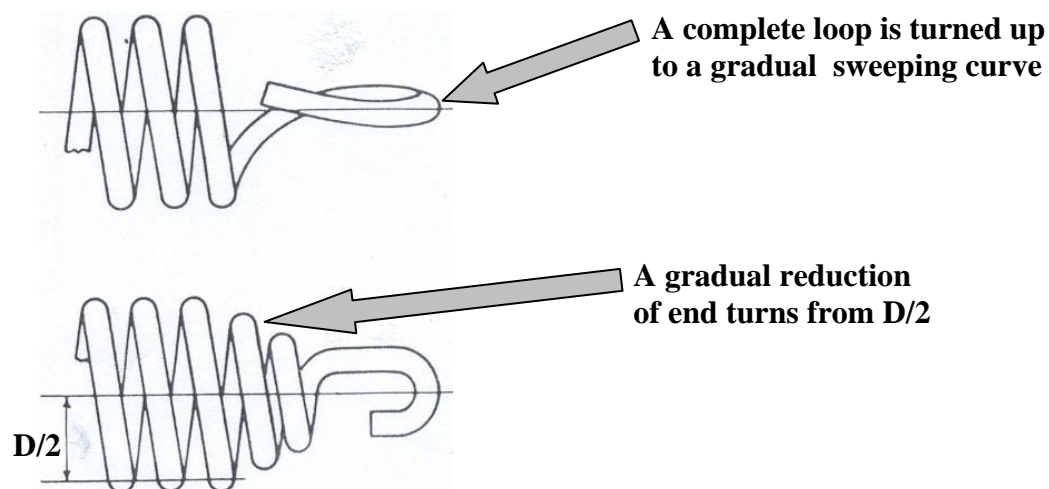Fig.7.2.7. The nomenclature for the extension spring is given below.

Body length, L_B : d (N + 1)
Free length, L   : L_B + 2 hook diameter.

here, N stands for the number of active coils. By putting the hook certain amount of stress concentration comes in the bent zone of the hook and these are substantially weaker zones than the other part of the spring. One should take up steps so that stress concentration in this region is reduced. For the reduction of stress concentration at the hook some of the modifications of spring are shown in Fig 7.2.8.

Extension spring

Fig 7.2.7

A complete loop is turned up to a gradual sweeping curve

A gradual reduction of end turns from D/2

Extension springs with improved ends

Fig 7.2.8

7.2.3 Buckling of compression spring

Buckling is an instability that is normally shown up when a long bar or a column is applied with compressive type of load. Similar situation arise if a spring is too slender and long then it sways sideways and the failure is known as buckling failure. Buckling takes place for a compressive type of springs. Hence, the steps to be followed in design to avoid buckling is given below.

Free length (L) should be less than 4 times the coil diameter (D) to avoid buckling for most situations. For slender springs central guide rod is necessary.

A guideline for free length (L) of a spring to avoid buckling is as follows,

$$L < \frac{\pi D}{C_e}\sqrt{\frac{2(E-G)}{2G+E}}$$

(7.2.8)

$$L < 2.57\frac{D}{C_e}, \text{for steel}$$, Where, C_e is the end condition and its values are given below.

$\underline{C_e}$ **end condition**

2.0   **fixed and free end**
1.0   **hinged at both ends**
0.707   **hinged and fixed end**
0.5   **fixed at both ends**

If the spring is placed between two rigid plates, then end condition may be taken as 0.5. If after calculation it is found that the spring is likely to buckle then one has to use a guide rod passing through the center of the spring axis along which the compression action of the spring takes place.

7.2.4 Spring surge (*critical frequency*)

If a load F act on a spring there is a downward movement of the spring and due to this movement a wave travels along the spring in downward direction and a to and fro motion continues. This phenomenon can also be observed in closed water body where a disturbance moves toward the wall and then again returns back to the starting of the disturbance. This particular situation is called surge of spring. If the frequency of surging becomes equal to the natural frequency of the spring the resonant frequency will occur which may cause failure of the spring. Hence, one has to calculate natural frequency, known as the fundamental frequency of the spring and use a judgment to specify the operational frequency of the spring.

The fundamental frequency can be obtained from the relationship given below.

Fundamental frequency $f = \frac{1}{2}\sqrt{\frac{Kg}{W_s}}$

$$f = \frac{1}{4}\sqrt{\frac{Kg}{W_s}}$$

One end free and other end on flat plate.
(7.2.10)

Where, **K** : **Spring rate**

W$_S$: **Spring**

**weight =** **$2.47\gamma d^2 DN$** (7.2.11)

and **d** is the wire diameter, **D** is the coil diameter, **N** is the number of active coils and **γ** is the specific weight of spring material.

The operational frequency of the spring should be at least 15-20 times less than its fundamental frequency. This will ensure that the spring surge will not occur and even other higher modes of frequency can also be taken care of.

A problem on spring design

A helical spring is acted upon by a varying load of 300 N to 900 N respectively as shown in the figure. The spring deflection will be around 15 mm and outside diameter of the spring should be within 48-50 mm.
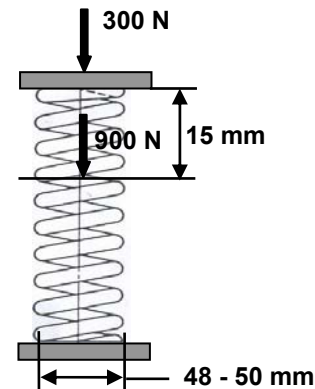
Solution

To design the spring for the given data, the most important parameter is the spring index. The spring index decides the dimension of the spring with respect to chosen wire diameter. Normally the spring index varies over a wide range from 3-12. For higher value of the spring index the curvature effect will be less, but relatively size of the spring and stress in the spring wire will increase. However, the effects will be some what opposite if the value of spring index is lower. Hence, it is better to start the iteration process with the spring index of 6-7.

Let us start the problem with spring index, C=6 and wire diameter, d=7 mm.

The above choice gives us a coil mean diameter, D =42 mm. Thereby, the outside diameter of the coil is 49 mm, which is within the given limit.
Computation of stresses:

The mean load $F_m = \dfrac{300+900}{2} = 600N$

stress amplitude, $T_a = \dfrac{900-300}{2} = 300 N$

Shear stress concentration factor, $k_s = 1 + \dfrac{1}{12} = 1.083$

Wahl correction factor, $k_w = \dfrac{4 \times 6 - 1}{4 \times 6 - 4} + \dfrac{0.615}{6} = 1.253$

So the value of mean shear stress, $\tau_m = 1.083 \times \dfrac{8 \times 600 \times 42}{\pi \times (7)^3} = 202.62 MPa$

and the value of stress amplitude, $\tau_a = 1.253 \times \dfrac{8 \times 300 \times 42}{\pi \times (7)^3} = 117.21 MPa$

Estimation of material properties:

As no specific use of the spring is mentioned in the problem, let us take Chrome Vanadium as the spring material. This alloy spring steel is used for high stress conditions and at high temperatures, it is also good for fatigue resistance and long endurance for shock and impact loads.

Ultimate strength of the material, $\sigma_{ut} = \dfrac{1790}{(7)^{0.155}} = 1324$ MPa

From the relationship of σ $_{ult}$ to τ$_y$ (yield point) and endurance limit, τ_e we find that

for chrome $\qquad \tau_y = \sigma_{ult} \times 0.51 = 675.2 MPa$ vanadium,

and $\qquad\qquad \tau_e = \sigma_{ult} \times 0.2 = 264.8 MPa$

From Soderberg equation,

$$\dfrac{\tau_a}{\dfrac{\tau_y}{FS} - \tau_m} = \dfrac{\dfrac{\tau_e}{2}}{\tau_y - \dfrac{\tau_e}{2}}$$

$$\dfrac{1}{FS} = \dfrac{\tau_m}{\tau_y} + \dfrac{\tau_a}{\tau_y}(\dfrac{2\tau_y}{\tau_e} - 1)$$

$$\dfrac{1}{FS} = \dfrac{202.62}{675.2} + \dfrac{117.21}{675.2}\left(\dfrac{2 \times 675.2}{264.8} - 1\right) = 1.01$$

$$\therefore FS \approx 1.00$$

Factor of safety, FS=1.0 implies that the design do not consider any unforeseen effect that may cause extra stresses in the spring. Normally in design of springs it is better to consider a factor of safety which should be in the vicinity of 1.3-1.5.

In order to increase the value of FS, in the next iteration, natural choice for the spring index, C is 5 and d = 8 mm. Because C=7 and d = 6 mm will lead to more stress on the wire and the value of FS will not improve.

With C=5 and d=8 mm and following the similar procedure as in previous iteration we have,

$$k_s = 1.1, \quad k_w = 1.311$$

Therefore,

$$\tau_m = \frac{1.1 \times 8 \times 600 \times 40}{\pi \times 8^3} = 131.3 MPa$$

$$\tau_a = \frac{1.311 \times 8 \times 300 \times 40}{\pi \times 8^3} = 78.24 MPa$$

Material properties:

$$\sigma_{ut} = \frac{1790}{(8)^{0.155}}$$

$$= 1297 \ MPa$$

Finally,

$$\tau_y = 661.4 \ MPa$$

$$\tau_e = 259.4 \ MPa$$

$$\therefore \frac{1}{FS} = \frac{131.3}{661.4} + \frac{78.24}{661.4}\left(\frac{2 \times 661.4}{259.4} - 1\right) = 0.684$$

$$FS = 1.46$$

The factor of safety obtained is acceptable. Therefore the value of spring index is 5 and corresponding wire diameter is 8mm.

Hence, mean spring diameter, D=40 mm.
Outer diameter of spring, D_o=40+8=48 mm, This value is within the prescribed limit.
Inner diameter of spring, $D_i = 32 \, mm$.

Spring rate, $k = \dfrac{900 - 300}{15} = 40 N / mm = 40 \times 10^3 N / m$

Once the value of stiffness is known, then the value of number of active turns, N of the spring is,

$$k = \frac{Gd^4}{8D^3N} \qquad \therefore N = \frac{80 \times 10^3 \times 8^4}{8 \times (40)^3 \times k} = 16$$

$$\therefore \delta \, max = \frac{8 \times 900 \times (40)^3 \times 16}{80 \times 10^3 \times 8^4} = 22.5 mm$$

In the above equation, G = 80000 MPa.

Spring Nomenclature:

Let us select the type of spring as squared and ground ends. For this type of spring the value of free length is,

$$L = L_S + \delta_{\max} + \delta_{allowance}$$

$$where,\ L_S = dN_T = 8.0 \times (16+2) = 144mm$$

$$\delta_{allowance} = 15\%\delta_{\max}$$

$$L = 18 \times 8 + 22.5 + 15\%\delta_{max} \approx 170mm$$

$$Pitch,\ p = \frac{L-2d}{N} = \frac{170-16}{16} = 9.625mm$$

Check for buckling:

We know that for steel,

$$L < 2.57\frac{D}{C_e} = 206mm$$

Here, for the given spring seat configuration, C_e = 0.5

The free length of the spring, 170 mm is less than the critical length for buckling, 206mm. Therefore the design is safe.

Check for critical frequency:

In order to find the critical frequency of the spring, the weight of the spring is to be first computed,

$$W_s = \frac{\pi d^2}{4}(\pi DN)(\gamma)$$

$$W_s = 2.47\gamma d^2 DN$$

$$W_S = 2.47 \times (8X10^{-3})^2 \times (40X10^{-3}) \times 16 \times 7800 \times 9.81$$
$$= 7.74N$$

Therefore,

The fundamental frequency of the spring (for both ends within flat plates),

$$f = \frac{1}{2}\sqrt{\frac{Kg}{W_s}} = \frac{1}{2}\sqrt{\frac{40 X 10^3 X 9.81}{7.74}} \approx 112.6 Hz$$

Safe frequency for design should be at least 20 times less than the fundamental frequency to take care of more number of harmonics. Therefore, the spring frequency for should be around 6 Hz.

Questions and Answers

Q1. Do the helical spring experience reverse loading? What is the loading type called when varying load acts on a helical spring?

A1. The helical spring experiences only repeated load. It cannot experience reverse loading, because the spring will lose contact with the end supports.

Q2. What modification in Soderberg diagram is required when it is used for design of helical springs?

A2. In the earlier Soderberg diagram, we have used in the design for varying loads on the machine member, had only stress amplitude in the endurance limit representation, since, endurance limit value was for complete reversed loading. Here, in spring design, we use endurance limit value for repeated loads only. Hence, we have both stress amplitude and mean stress value of equal magnitude, $\frac{\tau_e}{2}$. Therefore, the endurance limit representation in Soderberg diagram changes to $\frac{\tau_e}{2}, \frac{\tau_e}{2}$.

Q3. What should be the safe frequency of a helical spring?

A3. Safe frequency for design should be at least 20 times less than the fundamental frequency of the spring to take care of more number of harmonics.

References

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3. M.F Spotts, Design of Machine Elements, Prentice Hall India Pvt. Limited, 6th Edition, 1991.